

Third Semester B.Tech. Degree Examination, December 2015  
(2008 Scheme)

08.301 : ENGINEERING MATHEMATICS II (CMPUNERFTAHS)

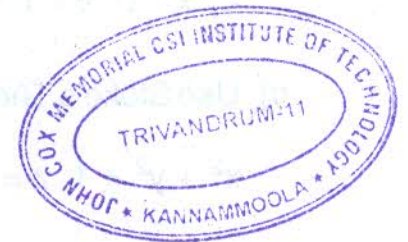
Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Evaluate  $\int_0^1 \int_0^{\sqrt{1+y^2}} \frac{1}{1+x^2+y^2} dx dy$ .
2. Find the area bounded by the parabolas  $y^2 = 4x$  and  $x^2 = 4y$ .
3. Find the workdone by the force  $\vec{F} = x\hat{i} - z\hat{j} - 2y\hat{k}$  in the displacement along the parabola  $y = 2x^2$ ,  $z = 2$  from  $(0, 0, 2)$  to  $(1, 2, 2)$ .
4. Obtain the half range sine series for  $e^x$  in  $0 < x < 1$ .
5. Expand  $f(x) = x^3$  in  $-\pi < x < \pi$  in a Fourier Series.
6. Find the Fourier Cosine transform of  $e^{-5x}$ .
7. Form the partial differential equation of all spheres of given radius  $a$  and whose centres lie on the  $xy$  – plane.
8. Solve  $z^2(p^2 + q^2 + 1) = C^2$ .
9. Find the particular Integral of  $(D^4 - D'^4)z = e^{x+y}$ .
10. How many Initial and boundary conditions are required to solve one-dimensional heat flow equation ? In steady state conditions derive the solution of one-dimensional heat equation.



P.T.O.



## PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

## Module – I

11. a) Change the order of integration in the integral  $I = \int_0^a \int_{x^2/a}^{2a-x} xy \, dy \, dx$  and evaluate it.
- b) Find the volume bounded by the paraboloid  $x^2 + y^2 = az$ , the cylinder  $x^2 + y^2 = 2ay$  and the plane  $z = 0$ .
- c) Evaluate  $\int_C (\cos x \sin y - xy) dx + \sin x \cos y \, dy$  by Green's Theorem where C is the circle  $x^2 + y^2 = 1$ .
12. a) Evaluate  $\iint_R x^2 \, dy \, dx$  where R is the two dimensional region bounded by the curves  $y = x$  and  $y = x^2$ .
- b) Verify divergence theorem for  $\vec{F} = x^2\mathbf{i} + z\mathbf{j} + yz\mathbf{k}$  taken over the cube bounded by  $x = 0, x = 1, y = 0, y = 1, z = 0$  and  $z = 1$ .
- c) Use Stoke's Theorem to evaluate  $\int_C yz \, dx + zx \, dy + xy \, dz$  where C is the curve  $x^2 + y^2 = 1, z = y^2$ .

## Module – II

13. a) Find the Fourier series expansion of the periodic function of period  $2\pi$ ,  $f(x) = x^2$  in  $-\pi < x < \pi$ . Hence deduce that

$$\text{i) } \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

$$\text{ii) } \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$

$$\text{iii) } \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$



b) Find the half range sine series for

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1. \end{cases}$$

c) Find the Fourier transform of

$$f(x) = \begin{cases} x, & |x| < a \\ 0, & |x| > a. \end{cases}$$

14. a) Using Fourier integral representation show that

$$\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda = \begin{cases} \frac{\pi}{2}, & 0 \leq x < 1 \\ \frac{\pi}{4}, & \text{for } x = 1 \\ 0, & x > 1 \end{cases}$$



b) If  $f(x) = x - x^2$  in  $-\pi < x < \pi$ . Deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

c) Find the Fourier sine transform of  $e^{-|x|}$ . Hence evaluate  $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx$ .



## Module – III

15. a) Solve  $p^2 - q^2 = x - y$ .
- b) Solve  $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y$ .
- c) Write the most general solution of the string equation, if the string (length  $l$ ) is fixed at both ends and is subjected to zero initial displacement and non zero initial velocity.
16. a) Solve  $\frac{\partial^2 z}{\partial x^2} + z = 0$  given that when  $x = 0$ ,  $z = e^y$  and  $\frac{\partial z}{\partial x} = 1$ .
- b) Solve  $z^4 p^2 - z^2 q = 0$ .
- c) The ends A and B of a rod 20 cm long have the temperature at  $30^\circ\text{C}$  and  $80^\circ\text{C}$  until steady. State conditions prevail. The temperature of the ends are changed to  $40^\circ\text{C}$  and  $60^\circ\text{C}$  respectively. Find the temperature distribution in the rod.
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